UK JUNIOR MATHEMATICAL CHALLENGE

THURSDAY 26th APRIL 2007

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

SOLUTIONS LEAFLET

This solutions leaflet for the JMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

1. **C** $0.1 + 0.2 + 0.3 \times 0.4 = 0.3 + 0.12 = 0.42$.

2. **D** The train arrived $5 + 42 = 47$ minutes after 17:40, that is at 18:27.

3. **B** Note that 7 divides 35, 49 and 7, so it divides 354970. So the remainder is 2.

4. **E** Of the options given, only 27, which is three less than a multiple of 5, namely 30, and three more than a multiple of 6, namely 24, has both of the properties in the question.

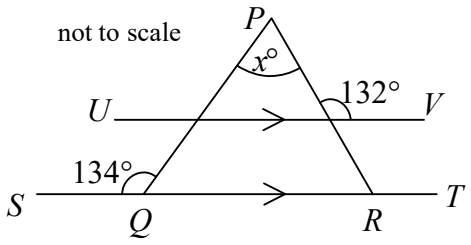
5. **E** The area of the large square may be considered to consist of thirteen equal squares (nine of which are shaded) plus eight 'half squares' and four 'quarter squares' (all of which are unshaded).
So the total unshaded area is $\left(4 + 8 \times \frac{1}{2} + 4 \times \frac{1}{4}\right)$ squares = 9 squares. Hence half of the large square is shaded.

6. **A** When put in their correct places on the number line, the order of the fractions is:
$-\frac{1}{3}, -\frac{1}{5}, -\frac{1}{7}, \frac{1}{6}, \frac{1}{4}$.

7. **D** If the top triangle is painted black, then any one of the three remaining triangles may also be painted black. Similarly, if the top triangle is painted white, then any one of the three remaining triangles may also be painted white. So there are six different ways.

8. **D** From the information, we see that Amy is to the left of both Ben and Chris. So the three are in the order Amy, Ben, Chris or the order Amy, Chris, Ben. So D is certainly true and the others are all false either in one case or in both.

9. **C** As ST is parallel to UV, $\angle PRT = 132°$ (corresponding angles).
So $\angle PRQ = 48°$ (angles on a straight line).
From the exterior angle of a triangle theorem, $\angle SQP = \angle QPR + \angle PRQ$, so
$x = 134 - 48 = 86$.



10. **D** The values of the five expressions are: A $\frac{3}{4}$; B $\frac{1}{4}$; C $\frac{1}{8}$; D 2; E $\frac{1}{2}$.

11. **A** The number of times each bar is used is: A 4; B 6; C 8; D 7; E 7.

12. **A** The total number of spots which the six ladybirds have is $6 \times 12 = 72$. So the number of spots which the pine ladybird has is $72 - (2 + 10 + 14 + 18 + 24) = 4$.

13. **D** If R is $(-5, 4)$ then $PQ = PR = 6$. If R is $(7, 1)$ or if R is $(-6, 1)$ then R lies on the perpendicular bisector of PQ (the line $y = 1$), so in both cases $PR = QR$. If R is $(7, -2)$, then $QP = QR = 6$. However if R is $(-6, -2)$, then $PQ = 6$, $QR = 7$ and $PR > 7$, so triangle PQR is scalene.

14. **E** The thickness of the line is 0.2 mm, that is 0.0002 m. So, in order to cover an area of one square metre, the length of the line would need to be $\dfrac{1}{0.0002}$ m, that is 5000 m.

15. **C** We consider the different possible choices from the top row. If 1 is chosen, then the options are 1, 5, 9 and 1, 6, 8 giving products 45 and 48 respectively. If 2 is chosen, the options are 2, 4, 9 and 2, 6, 7 giving products 72 and 84 respectively. Finally, if 3 is chosen, the options are 3, 4, 8 and 3, 5, 7 giving products 96 and 105. So 105 is the maximum.

16. **B** The six marked angles, together with the six interior angles of the two triangles, comprise all of the angles around five separate points. So the required sum is $(5 \times 360 - 2 \times 180)° = 1440°$.

17. **C** The only possible cubes have edge size 1 or 2. It takes 8 of the former to replace one of the latter, so William needs to cut as many cubes of edge size 2 as possible, namely 3. The number of one inch cubes, therefore, is $2 \times 3 \times 6 - 3 \times 8$, that is 12. So the smallest number of cubes is $3 + 12 = 15$.

18. **B** The hundreds column shows us that $J = 1$ or 2. [We can't carry more than 2 from the units to the tens; and 2 plus the biggest feasible values 7, 8, 9 for the three letters is only 26.] The units column shows that $J + M$ is a multiple of 10 and it can't be 0 (or else $J + M = 0$); so $J + M = 10$ and $M = 9$ or 8 respectively. Also, the sum of the units column is $10 + C$, so there is exactly 1 to carry to the tens column. The tens column now tells us that $J + C + 1 = 10J$. So $J = 2$ is not possible and therefore $J = 1$, $C = 8$ and $M = 9$.

19. **A** If the semicircle with diameter PQ is rotated through 180° about Q, the new shape formed has the same area as the original shape. It consists of a semicircle of diameter 6 cm and a semicircle of diameter 2 cm. So its area is $\left(\frac{1}{2} \times \pi \times 3^2 + \frac{1}{2} \times \pi \times 1^2\right)$ cm^2, that is 5π cm^2.

20. **E** Range Hill scored only three points in the match and these were scored in the second half. They represent 10% of the total points scored. As Boarwarts Academy also scored three points in the second half, the proportion of points scored after halftime was 20%, that is $\frac{1}{5}$.

21. **B** Let the list be $3,\ 4, a, b, c, d, e, f, g, h$. We can see that $c = 3$ and $e = 4$. So the list now reads $3,\ 4, a, b, 3, d, 4, f, g, h$. Now, the only pairs of letters two apart from each other are a, d and d, g. Therefore $d = 2$ and the list is $3,\ 4, a, b,\ 3,\ 2,\ 4, f, g, h$. The only pair now one apart are f, h. The list is 3, 4, a, b, 3, 2, 4, 1, g, 1. Now a, b are the only pair zero apart. So $a = b = 0$ and $g = 2$.

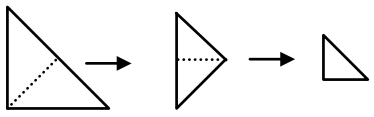
22. **D** Four of the given values for d may be rejected since $143 = 11 \times 13$; $153 = 3 \times 51$; $567 = 3 \times 189$; $183 = 3 \times 61$. However, 173 and 577 are both prime, so $d = 7$.

23. **E** Let the length of the longer side of each rectangle be l. Then the length of each shorter side is $l - x$. So $y = l + l - x$ and hence $l = \frac{1}{2}(y + x)$.

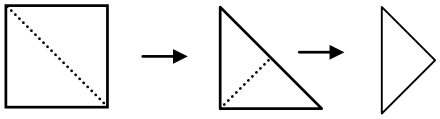
24. **C** Pages 1 to 9 inclusive require 9 digits; pages 10 to 99 inclusive require 180 digits. So, in total, 189 digits are required to number all of the pages before the three-digit page numbers commence with page number 100. This leaves 663 digits, so the last page in the book is the 221st page which has a three-digit number, namely page 320.

25 **B** Imagine unfolding the final triangle once. Then one edge of the final triangle is inside the new shape obtained; and the other two triangle edges have 'mirror image' copies. So the new shape has at most 4 edges. After unfolding once more, one of these edges is now on the inside; and the remaining edges get mirror images again. So the shape obtained (the original shape) has no more than 6 edges. The diagrams below show that 3, 4, 5 and 6 sides are all possible.

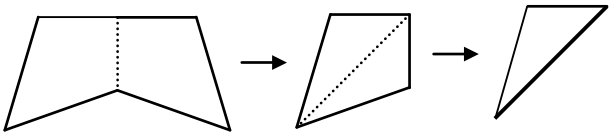
Triangle:



Square:



Pentagon:



Hexagon: